B.Sc. 4th Semester (Honours) Examination, 2021-22

PHYSICS

Course ID: 42411 **Course Code: SH/PHS/401/C-8/T-8**

Course Title: Mathematical Physics III

Time: 1 hour 15 minutes **Full Marks: 25**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words
As far as practicable

Section-I

1. Answer *any **five** questions*: 1×5=5

a) Show that in vector space V, any set of vectors containing the zero vector is linearly independent.

b) Define a group.

c) Explain isomorphic vector spaces.

d) If ψ and ϕ are two orthogonal vectors in a Euclidean space then

$$\|\psi + \phi\|^2 = \|\psi\|^2 + \|\phi\|^2$$

e) What is the importance of Fourier transform?

f) Write down the expression $F\{x^n f(x)\}$, F stand for Fourier Transform.

g) Using differentiation of Laplace transform determine the value of $L\{te^{at}\}$.

h) State Caley Hamilton theorem.

Section-II

2. Answer any ***two*** questions: 5×2=10

(a) (i) Determine whether the vectors u = (1, 1, 2), v = (1, 0, 1), and w = (2, 1, 3) span the linear vector space R^3. (ii) Let T: V → U be a linear transformation. What do you mean by the kernel of T and the range of T? 3 + 2

(b) Using Parseval's identity show that $\int_0^{\infty} \frac{dx}{(x^2+1)^2} = \frac{\pi}{4}$ 5

(c) Find the Fourier sine transformation of e^{-ax} (a>0) and then show that $\int_0^{\infty} \frac{x \sin kx}{a^2+x^2} dx = \frac{\pi}{2} e^{-ak}$ where k>0. 3+2

(d) (i) Given that $L(t^3) = \frac{6}{s^4}$; find $L(t^6)$.

(ii) Find the inverse Laplace transform of $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$ 2+3

Section-III

3. Answer any ***one*** question: 10×1=10

(a) (i) If L[f(t)] = F(s), then show that L $[t^n f(t)]$ = $(-1)^n \frac{d^n}{ds^n}[F(s)]$

(ii) If L{f(t)} = F(s) then prove L{f(at)} = $\frac{1}{a} F(\frac{s}{a})$

(iii) Find the Fourier cosine and sine transform of x^{m-1} (m>0) 3+2+5

(b) (i) Consider the differential equation: $\frac{dy}{dt} + ay = e^{-bt}$ with the initial condition y (0) = 0. Then find the Laplace transform Y(s) of the solution y(t).

(ii) Let u (x, t) denote the temperature at distance x and time t. Solve the heat flow equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ $x \geq 0, t \geq 0$ under the boundary condition u = u0 when x = 0, t > 0 and the initial condition u = 0 when x > 0, t = 0.

(iii) Show that a necessary and sufficient condition for a matrix A to be diagonalisable is that A has all linearly independent eigenvectors. 4+4+2